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CORRELATION AND SPECTRUM MODELING IN BACKSCATTERED FIELD FOR NONSTATIONARY ONE-DIMENSIONAL STATISTICAL PROBLEM

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In proposed paper we continue the investigations have been begun earlier [1,2], devoted to statistical modeling of pulses and signals scattering on the inhomogeneously-layered fluctuating medium. The studying of the problem is carried out directly in the framework of spatial-time representation on the basis of exact solving of wave equation involving variable coefficients and correspondent boundary and initial conditions [3]. The possibility of this boundary value problem exact solution obtaining for every single realization allows to study the behaviour of backscattered field statistical characteristics for different type-form original pulses, incident on the medium with sound velocity fluctuations. Comparison of the exact simulation results with the approximate ones known earlier [4,5] demonstrates the number of differences and it allows to determine the applicability region of the approximate description.

SETTING UP THE PROBLEM

Mathematical description of the considering problem of the medium pulse probing is the following stochastic boundary value problem for wave equation:

$$\frac{\P^2 G(z,L,t)}{\P z^2} - \frac{1}{c^2(z)}\frac{\P^2 G(z,L,t)}{\P t^2} = 0,$$
$$\left(\frac{\P}{\P z} + \frac{1}{c_0}\frac{\P}{\P t}\right)G(z,L,t)\Bigg|_{z=L} = \frac{2}{c_0}\frac{\P}{\P t}q(t)\,, \quad \left(\frac{\P}{\P z} - \frac{1}{c_0}\frac{\P}{\P t}\right)G(z,L,t)\Bigg|_{z=L_0} = 0. \qquad (1)$$

Hereafter the pulse as the unit Heaviside function $q\,[(z-L)/\tilde{n}_0 + t\,]$ is incident on the layer boundary L of the inhomogeneous medium $(L_0\,,L)$ having the fluctuating sound velocity profile $c(z)$. The incidence of the pulse takes place at the time moment $t = +\,0$ from the half-space $z > L$. For $t > 0$ in this half-space $z > L$ the backscattered field $R[(z-L)/\tilde{n}_0 - t\,] = G(L,L,t) - q\,(t)$ appears. It contains the information about the pulse scattering processes on the random medium inhomogeneities. In papers [1,2] the behaviour of backscattered field statistical moments have been analyzed for incident pulses of various duration. It was shown that the scattering has nonstationary character and the results for its quantity description were represented. In present paper we focus attention on the results of random field *R(t)* correlation function and power spectrum density investigation.

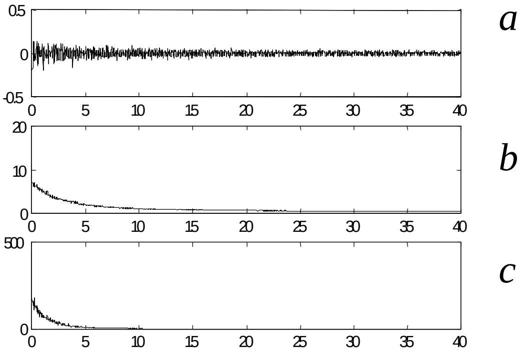
RESULTS OF SIMULATION

Let's set the profile fluctuations in the form of Gaussian random process $c(z) = c_0\,(1+e(z))$ having zero mean value and correlation function $<e(z)e(z')> = s^2_\varepsilon\,exp(-|z-z'|/l)$, where intensity of fluctuations $s^2_\varepsilon <<1$, and correlation radius l is the least spatial scale of the problem. Let's introduce into consideration the parameters of referencing to some basic narrow-band signal of the carrier frequency W : $D(W)$ is the diffusion coefficient for this frequency [5], $T = D^{-1}/c_0$ is the correspondent time scale of diffusion. If $c(z)$ is the determinate function, then the problem (1) solution at $z = L$, that is the field $R(t) = R_L(t)$, can be written in the analytic form while use piece-wise grid approximation of the profile $c(z)$ by the certain form functions with any required accuracy [1-3]. For lack of place here we don't present this expression, that was used in the process of field calculations for every realization of the random function $e(z)$, assigned by its ensemble, and pay our attention directly to simulation results. Further, as it has been earlier [2,6], only normalized time variables will be taken into consideration: $W = WT$, $t = t/T$, $x(z) = T^{-1}\int_{L_0}^{z} dz\, c^{-1}(z)$, $h = x(L)$, $R(t) = R_L(t)/T$. The following values of the parameters have been chosen: $W = 100$, $h = 20$, $s^2_\varepsilon = 0{,}025$. Two limiting cases of the pulses incident on the random medium are considered: the pulse of very long duration $h > h$, as the Heaviside function is, and very short time pulse $h << 1 < h$ (it is the delta-pulse model). These cases are principally different from the situations, for which the efforts have been undertaken earlier by another researches to obtain the asymptotic results by the approximate methods [4,5]. In figures below the behaviour of backscattered field statistical moments $<R(t)>$, $<R^2(t)>$, $<R^4(t)>$ (figs.1,2), correlation functions and power spectrum is represented for indicated durations h of the incident pulse. Figs.1,2 demonstrate the pronounced nonstationarity of backscattering process, that has been established by us earlier [1,2]. In this paper we focus attention on the behaviour of correlation functions and power spectrum densities of backscattered field. For the general case of nonstationary process they can be written as

$$Y(t,d) = <R(t)\, R(t+d)> = \int_{-\infty}^{\infty} dw_1 \int_{-\infty}^{\infty} dw_2 < \hat{R}(w_1)\hat{R}^*(w_2) > exp\{iw_1 t - iw_2 (t+d)\} ,$$

$$S_w(t) = (2p)^{-1} \int dd\, Y(t,d) e^{-iwd} = e^{-iwt} \int_{-\infty}^{\infty} dw_1 < \hat{R}(w_1)\hat{R}^*(w) > exp\{iw_1 t\}$$

correspondently. In fig.3 the behaviour in time of the normalized correlation function of backscattered field is represented for the interval d Î [-2,2] in the case that q - pulse is incident on the medium layer. It is seen, that in the region of nonstationarity an evident decrease of the correlation function amplitude $Y(t,0)$ (because $Y(t,0) = <R^2(t)>$) takes place as well as both the relative width of correlation coefficient of backscattered field increases and its small-scale fluctuations more and more disappear. This increase of correlation means the narrowing of power spectrum of the process. In fact, Fig. 4 illustrates this statement for $f = \tilde{w}/2p$, where f Î [0, 250]. During the increase of observation time t the power spectrum high frequency components in backscattered field disappear and the spectrum more and more displaces itself to low frequency region. Simulteneously its amplitude near by $f \sim 0$ increases. These low frequencies form the infinite coda of q - pulse and they penetrate to the great depth of the medium without scattering. Namely by these components is caused the power law of decaying of the backscattered field second moment, have been established in [4-6]. Thus in this problem the scattering process stationarization for the Green function case is not takes place. One can speak only about some quasi-stationary regime for the times $t \sim 30 \div 40$, correspondent to the going out of $Y(t,0) = <R^2(t)>$ to the level $\cong 0{,}7 \cdot 10^{-3}$ [6]. Now we pay attention to the delta-pulse model. Here we don't present the correspondent correlation functions for lack of place. They have the similar to previous ones form, but with the considerably lesser relative width only. The function of the process power spectrum is shown in fig. 5. Its characteristic feature is the decay of the spectrum level during the time (see fig.1) and its gradual levelling. For the long time the dip of the level is appeared for the moderate and high frequencies. In this case for the region $f \sim 0$ we have very small values of the spectrum amplitude. It indicates on the absence in backscattered field of the components with such frequencies, because they pass through the medium layer without scattering. More high frequencies gradually egress themselves from the medium backward, forming the backscattered field. Practically all the energy of delta-pulse egresses backward to the observation time moment $t \sim 20$. In conclusion let's note, that for the considered nonstationary region of the backscattering process the approximate results of papers [4,5], giving fairly long asymptotics of the field $R(t)$ moments, are unapplicable. Besides these results have also small applicability to the analysis of such type incident pulses, as the delta and q - pulses, because they were obtained for the approximation of narrow-band signal, incident on the random medium.



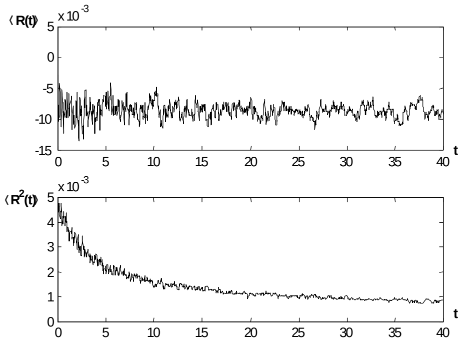
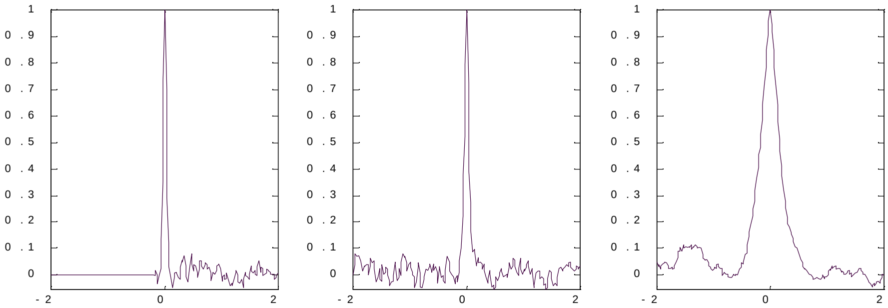


Fig.1. The t - behaviour of backscattered field statistical moments while d - pulse is incident on the random medium:

Fig.2. The same as in the fig.1, but for incident q - pulse.



a - $\langle R(t)\rangle$,b - $\langle R^2(t)\rangle$, c - $\langle R^4(t)\rangle$.

Fig.3. Time dependence of correlation coefficient for incident q - pulse. From right to left: t = 0,2 ; t = 2 ; t = 35 .

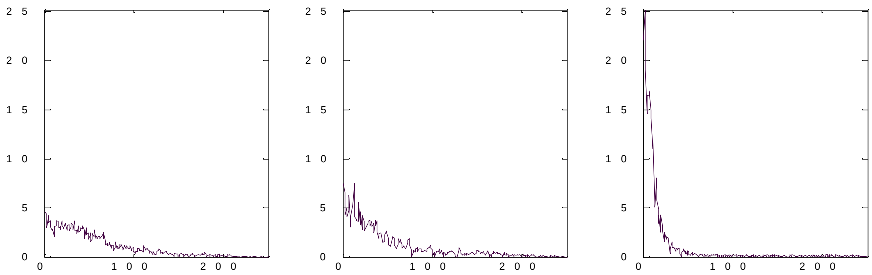


Fig.4. Power spectrum density behaviour correspondent to fig.3 .

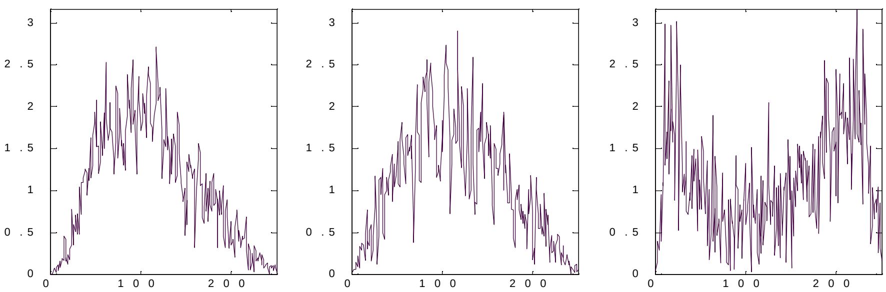


Fig.5. Power spectrum density behaviour for incident d - pulse. From right to left: t = 0,2 ; t = 2 ; t = 35 .

R E F E R E N C E S

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