V.N.Alexeyev, S.À.Rybak

WAVE PROCESSESES IN MEDIA CONTAINING COMPACT OSCILLATORS

N.N. Andreyev Acoustics Institute
ul. Shvernika 4, Moscow, 117036 Russia
Phone (095)126-9072; Fax: (095) 126-8411;
E-mail: rybak@akin.ru

The great number of works is devoted to a question of sound waves propagation in a liquid containing gas bubbles, in which for the decision of the formulated task the various methods and approaches were used. Among these works we shall mention one of them [1], in which the rather simple model equation of resonant dispersion was offered. This equation, having a known generality, intended for the description of intensive sound propagation in various media containing compact resonators. The obtained equation was used with success, in particular, for study of nonlinear processes taken places in liquids with bubbles. Bubbles media occupy the special position in nonlinear acoustics, representing if not unique, the most real example of the nonlinear media having sharply expressed resonant dispersion.

However spherical cavities can be met not only in liquids, but also in such media, in which at their deformations arise not only normal, but also shear tension. Really such bubbles media meet also frequently as in alive, and lifeless nature. The gas cavities can exist in many biological tissues having the shear module. Quite often the similar media with bubbles and with the shear equal to zero module are used and in many technological processes on plants too. It is possible to mention here, for example, polymers, very viscous liquids, liquid with pollution and etc.

Below we shall consider radial oscillations of a spherical cavity filled with gas and located in viscoelastic media. In this case the equation describing media dynamics will look as well as in a solid

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\frac{\partial\sigma_{ik}}{\partial x_k} \quad . \tag{1}$$

However in contrast to solid the tension tensor will be related with the deformations tensor not with the help of the usual Hook law, but by more complex manner. Using the generalized Oldroid model, displacement particle relation with the deformations tensor we shall write down in such kind

$$\sigma_{ik}+\tau_1\frac{\partial\sigma_{ik}}{\partial t}=\mu\left(1+\tau_2\frac{\partial}{\partial t}\right)\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}-2\delta_{ik}\operatorname{div}\mathbf{u}\right)+K\left(1+\tau_3\frac{\partial}{\partial t}\right)\delta_{ik}\operatorname{div}\mathbf{u} \quad . \tag{2}$$

Here τ_1, τ_2 è τ_3 are so-called relaxation times. Let's notice, that in case of zero equality of all times τ_1, τ_2 è τ_3 the equation (2) is reduced to the classical Hook equation identically. In the same case, when $\tau_2 \neq 0$ è $\tau_3 \neq 0$, à $\tau_1 = 0$ the equations (1) and (2) describe dynamics of a usual solid, but with taking into account the internal friction. At that times also quantities $\eta = \mu\tau_2$ è $\zeta = K\tau_3$ play a role of friction coefficients similar to those, which are used in the theory of a viscous liquid [2]. In the theory of viscoelastic media using of the equation (2) with such parameters corresponds to the widespread Voigt model [3,4]. In the other limiting case, when $\tau_2 = \tau_3 = 0$, but $\tau_1 \neq 0$, the equation (2) describes viscoelastic media within the framework of Maxwell model. Let's notice, that in the classical theory of elasticity this model corresponds to the description of strongly viscous liquids [2]. On the other hand it is necessary to note, that in a case monochromatic cavity oscillations the using of the general equation (2) practically is equivalent to using of the common Hook law. Formally the introduction of the additional operators of time differentiation in the equation (2) in this case leads to renormalization of elastic moduli, which acquire now imaginary components. Thus the modules replacement occurs by the following rule:

$$K \to K\frac{(1-i\omega\tau_3)}{(1-i\omega\tau_1)} \;\; ; \;\; \mu \to \mu\frac{(1-i\omega\tau_2)}{(1-i\omega\tau_1)} \quad . \tag{3}$$

We shall consider now fragmentary the problem of sound scattering by single gas bubble, considering, that the cavity has an elastic shell or film of thickness h, which is described also within the framework of the equations (1) and (2). Thus we shall suppose, that elastic moduli, and other shell parameters can differ from the appropriate parameters of external medium. The standard procedure of solving the equations (1) and (2) together with the equations for internal region filled with gas, and appropriate boundary conditions results that the expression for sound scattering amplitude can be written down in a usual kind

$$f = \frac{\overline{R}}{\left(\omega_0 / \omega\right)^2 - 1 - i\delta} \quad . \tag{4}$$

However expressions for the appropriate effective parameters in the given formula appear to be different. So with the accuracy to a sign the limiting value for the real part of scattering amplitude becomes equal to the following expression [5]

$$R^* = \overline{R}\,\frac{\left(\rho_2 \overline{R} + \rho_2 h\right)}{\left(\rho_2 \overline{R} + \rho_1 h\right)} \quad .$$

(5)

The bottom indexes here and further concern in the case 1 to parameters of shell substance and in the case 2 - to external viscoelastic medium.

From the given expression (5) it follows, that when the shell is absent at the bubble, it really converges, as well as in a liquid, to average bubble radius. As if to resonant frequency, the simple calculations show, that expression for ω_0 in this case gets the following view [5]

$$\omega_0 = \frac{1}{\overline{R}} \sqrt{\left(4\mu_1 + 4\left(\mu_2 - \mu_1\right)\frac{\overline{R}^3}{\left(\overline{R} + h\right)^3} + 3\gamma_g P_g\right)\left(\frac{\overline{R} + h}{\rho_2 \overline{R} + \rho_1 h}\right)} \quad . \tag{6}$$

In limiting cases of gas bubbles, taking place in an ideal liquid and empty cavity in a solid, the expression (6) tends to the known limiting values. Thus it is necessary to note, that in a case of viscoelastic medium, but at shell absence at bubble, the resonant frequency square is equal to the sum of frequencies squares determined by the appropriate expressions for bubble gas in a liquid [5] and for an empty cavity in a solid [2]. This surprisingly simple result was received, in particular, in earlier works [6,7].

As to losses of oscillatory energy, that, as well as in a case of bubbles in a liquid, they consist of three components: $\delta = \delta_a + \delta_v + \delta_t$. Thus the acoustic losses appear now so [5]

$$\delta_a = k_2 \overline{R}\,\frac{\rho_2\left(\overline{R} + h\right)}{\rho_2 \overline{R} + \rho_1 h} \quad . \tag{7}$$

The expression (7) is formal coincides with similar expression for acoustic losses in case of the ideal liquid which has been written down as $\delta_a = k\overline{R}$. But in this case its effective value R^* is determined by the formula (5) instead of average bubble radius. The wave number $k_2 = \omega / c_{21}$ is defined now by speed of propagation of longitudinal waves in external viscoelastic medium - 2.

The losses connected to internal friction in viscoelastic medium, are obtained with the help of the following formula [5]

$$\delta_v = 4\,\frac{\eta_1\left(\overline{R} + h\right)^3 + \left(\eta_1 - \eta_2\right)\overline{R}^3}{\omega \overline{R}^2\left(\rho_2 R + \rho_1 h\right)\left(\overline{R} + h\right)^2} \quad . \tag{8}$$

This expression is easy to receive, using the formulae (4) and (6) and assuming, that the shear modulus carries now complex character according to the formula (3). Thus we shall consider, that the imaginary part of the shear modulus is expressed through the friction coefficient according to definition $\mathrm{Im}\left(\mu\right) = -\omega\eta$. Let's notice, that at film absence the received expression coincides with the similar formula for viscous losses for gas bubble, situated in a liquid.

Expression for thermal losses in view of its extreme complexity we shall not cite here. However in connection with some features of thermal processes occurring in a liquid in a vicinity gas bubbles, we shall make some remarks. First, as it is visible from the given formulas (7) and (8), at the large sizes of a cavity the acoustic losses prevail, and for bubbles of the small sizes the viscous losses are essential. From the literature it is known [8], that for gas bubbles, situated in a liquid and in intermediate area of their sizes, the thermal losses of acoustic energy are most essential. Secondly, we shall remind, that the thermal losses are defined by a temperature gradient arising in medium at a sound wave passage through it. Thus the adiabatic change of temperature in each phases are defined with the help of relation $\delta T = (\alpha T / \rho c_p)\delta p$, where α - thermal expansion coefficient, c_p - specific heat of substance at constant pressure, and δp - change of pressure caused by a sound propagation. As for gas $\alpha T \approx 1$, and for a liquid $\alpha T \ll 1$, and density of the condensed phase practically on three order is more than density of gas. Therefore adiabatic change of temperature in a gas phase considerably exceed corresponding change in a liquid. Length of a thermal boundary layer is defined by the minimal values from three characteristic lengths: $\overline{R}, \lambda$ è $\sqrt{2\chi/\omega}$, where $\chi = \kappa / \rho c_p$ - coefficient of thermal conductivity. Coefficient of thermal conductivity for many condensed substances appears almost on two order less than appropriate factor for gas. Nevertheless estimations of temperatures gradients arising in twophase medium show, that they are determined by gas component for the most part. Practically for finding of temperature distribution in the bubble vicinity, situated in a liquid, it is enough to solve a thermal task only for gas component.

Our remarks concerning thermal losses in viscoelastic medium are reduced to two statements. First, as well as in a liquid, the thermal losses are most essential in some and enough large intermediate area of size changes of gas cavities and carry determining character. Secondly, for many viscoelastic media the expression for thermal losses practically differs little from the appropriate expression received for radial oscillations of gas bubbles, situated in a liquid. This statement is based in the fact that practically for all condensed substances $\alpha T \ll 1$ in contrast to gas and density of the condensed substance is much greater then the gas density. In this case temperature change under sound action is most essential in a gas phase and in the most practically important cases the gradient of temperature is determined by an interior of a gas cavity.

REFERENCES

1. *Rybak S.A, Skrynnikov Yu. I.* Nonlinear Waves in Resonant Dispersion Media. - Nonlinear world, IV Int. Workshop, Kiev, 1989, Proceedings, World Scientific, Singapore, p.664-670.
2. *Landau L.D. and Lifshits Å.Ì.* The theory of elasticity. Moscow, Nauka, 1987.
3. *Michajlov I.G., Solov'ev V.A., Sirnikov Yu.P.* Molecular acoustics. Moscow, Nauka, 1964. (In Russian).
4. Physical Principles of Medical Ultrasonics. Hill, C.R., Ed., New York, 1986.
5. *Alexeyev V.N., Rybak S.A.* Gas bubble oscillations in elastic media. Acoustical Physics., 1999, vol.45, ¹ 5, pp.535-540.
6. *Andreeva I.B.*. Sound scattering by swimming-bladders of fish in deep-water sound-scattering layers of ocean. Akust. Zh., 1964, vol.10, ¹ 1, pp. 20-24. (In Russian).
7. *Alexeyev V.N., Rybak S.A.* The behavior of gas bubbles in insonated biological tissues. Acoust. Phys., vol. 44, ¹ 2, pp. 243-247.
8. *Plesset M.S., Hsieh D.Y.* Theory of gas bubble dynamics in oscillating pressure fields. Phys. Fluids, 1960, vol. 3, ¹ 6, pp.882-892.